Engineering Calculus I - MAC 2281 - Section 002

QUIZ II

First Name: Last Name:

1. (6 points)

Compute the following limits. Show all work and **state any theorems or special limits used**.

- $\lim_{x\to 2} \frac{4x-x^3}{2-x} \underset{\text{plug-in}}{=} \frac{4\cdot 2-2^3}{2-2} = \frac{\text{“}0\text{”}}{0}$

  Hence we need to do more work to compute the limit:

  $$\lim_{x\to 2} \frac{4x-x^3}{2-x} = \lim_{x\to 2} \frac{x(4-x^2)}{2-x} = \lim_{x\to 2} \frac{x(2-x)(2+x)}{2-x} = \lim_{x\to 2} x(2+x) \underset{\text{plug-in}}{=} 2(2+2) = 8$$

- $\lim_{x\to \pi} \frac{3(x-\pi)}{\sin(x-\pi)} \underset{\text{plug-in}}{=} \frac{3(\pi-\pi)}{\sin(\pi-\pi)} = \frac{\text{“}0\text{”}}{0}$

  Hence we need to do more work to compute the limit.
  We use the fact that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ (Special Limit)

  Note that

(*) $$\lim_{x\to 0} \frac{x}{\sin(x)} = \lim_{x\to 0} \frac{1}{\frac{\sin(x)}{x}} = \frac{\lim_{x\to 0} 1}{\lim_{x\to 0} \frac{\sin(x)}{x}} \underset{\lim_{x\to 0} \frac{\sin(x)}{x} = 1}{=} \frac{1}{1} = 1$$

  Then

  $$\lim_{x\to \pi} \frac{3(x-\pi)}{\sin(x-\pi)} = 3 \lim_{x\to \pi} \frac{x-\pi}{\sin(x-\pi)} \underset{\substack{\text{set } t = x-\pi \\ \text{when } x\to\pi, \text{ we have } t\to 0}}{=} 3 \lim_{t\to 0} \frac{t}{\sin(t)} \underset{\text{by (*)}}{=} 3\cdot 1 = 3$$

2. (4 points) State the Squeeze Theorem.

If $f(x) \leq g(x) \leq h(x)$ for all x near a (except possibly at a)

and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$,

then $\lim_{x \to a} g(x) = L$