Engineering Calculus I - MAC 2281 - Section 002

QUIZ IV

First Name: Last Name:

1. (2 points)

State the Product Rule.

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

or

$$\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$$

2. (5 points)

Prove the Product Rule.

Want to show: $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$

Start the proof by stating what you want to prove

$$[f(x)g(x)]' = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) \overbrace{- f(x+h)g(x) + f(x+h)g(x)}^{=0} - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \left[f(x+h)\frac{g(x+h) - g(x)}{h} + g(x)\frac{f(x+h) - f(x)}{h} \right]$$

$$= \lim_{h\to 0} f(x+h)\frac{g(x+h) - g(x)}{h} + \lim_{h\to 0} g(x)\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} f(x+h) \cdot \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} + \lim_{h\to 0} g(x) \cdot \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= f(x)g'(x) + g(x)f'(x)$$

$$= f'(x)g(x) + f(x)g'(x)$$

3. (3 points)

Compute $\frac{d}{dx}\left[\frac{2x}{x^2+\cos(x)}\right]$. Show sufficient work to **communicate** your process.

$$\frac{d}{dx}\left[\frac{2x}{x^2+\cos(x)}\right] \underset{\text{quotient rule}}{=} \frac{\frac{d}{dx}[2x]\cdot(x^2+\cos(x)) - 2x\cdot\frac{d}{dx}\left[x^2+\cos(x)\right]}{(x^2+\cos(x))^2}$$

$$= \frac{2(x^2+\cos(x)) - 2x(2x-\sin(x))}{(x^2+\cos(x))^2}$$

$$= \frac{-2x^2+2\cos(x)+2x\sin(x)}{(x^2+\cos(x))^2}$$

Stop here to avoid
mistakes in algebra