Engineering Calculus I - MAC 2281 - Section 002

QUIZ V

First Name: Last Name:

In the following exercises, show sufficient work to **communicate** your process.

1. (3 points)

Let $y = \cos^2(e^x)$. Compute $\frac{dy}{dx}$.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left[\cos^2(e^x)\right] \\
&= \frac{d}{dx}\left[(\cos(e^x))^2\right] \\
&\underset{\text{chain rule}}{=} 2\cos(e^x)\cdot\frac{d}{dx}\left[\cos(e^x)\right] \\
&\underset{\text{chain rule}}{=} 2\cos(e^x)\cdot(-\sin(e^x))\cdot\frac{d}{dx}\left[e^x\right] \\
&= 2\cos(e^x)\cdot(-\sin(e^x))\cdot e^x \\
&= -2e^x\cdot\cos(e^x)\cdot\sin(e^x)
\end{aligned}$$

2. (3 points)

Let $x^2y + \pi^{87} = \tan(y)$. Compute $\frac{dy}{dx}$.

We use implicit differentiation:

$$\frac{d}{dx}\left[x^2y + \pi^{87}\right] = \boxed{\frac{d}{dx}\left[\tan(y)\right]} \quad \text{chain rule}$$

$$\text{product rule} \quad \boxed{\frac{d}{dx}\left[x^2y\right]} + \boxed{\frac{d}{dx}\left[\pi^{87}\right]} = \sec^2(y)\cdot\frac{dy}{dx}$$

$$\pi^{87} \text{ is a number} \quad \frac{d}{dx}\left[x^2\right]\cdot y + x^2\cdot\frac{dy}{dx} + 0 = \sec^2(y)\cdot\frac{dy}{dx}$$

$$\begin{aligned}
2xy + x^2\cdot\frac{dy}{dx} &= \sec^2(y)\cdot\frac{dy}{dx} \\
x^2\cdot\frac{dy}{dx} - \sec^2(y)\cdot\frac{dy}{dx} &= -2xy \\
\left(x^2 - \sec^2(y)\right)\cdot\frac{dy}{dx} &= -2xy \\
\frac{dy}{dx} &= \frac{-2xy}{x^2 - \sec^2(y)}
\end{aligned}$$

3. (4 points)

Let $f(x) = x^{\cos(x)}$. Compute $f'(x)$.

We use logarithmic differentiation:

$$y = x^{\cos(x)}$$

$$\ln(y) = \ln\left(x^{\cos(x)}\right)$$

$$\ln(y) = \cos(x) \cdot \ln(x)$$

chain rule $\boxed{\frac{d}{dx}[\ln(y)]} = \boxed{\frac{d}{dx}[\cos(x) \cdot \ln(x)]}$ product rule

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}[\cos(x)] \cdot \ln(x) + \cos(x) \cdot \frac{d}{dx}[\ln(x)]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = -\sin(x) \cdot \ln(x) + \cos(x) \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y \cdot \left(-\sin(x) \cdot \ln(x) + \frac{\cos(x)}{x}\right)$$

$$\frac{dy}{dx} = x^{\cos(x)} \cdot \left(-\sin(x) \cdot \ln(x) + \frac{\cos(x)}{x}\right)$$