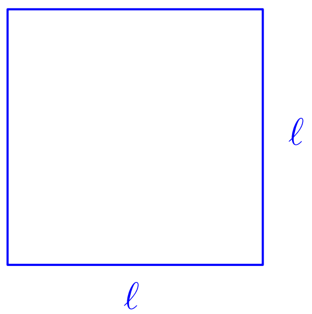
Engineering Calculus I - MAC 2281 - Section 002

QUIZ VI

First Name: Last Name:

1. (5 points)

A square is expanding at a rate of 2 cm^2/s (maintaining its square shape). How fast is each side expanding when the area is 25 cm^2 ?



- t time s
- ℓ side of the square cm
- A area of the square cm^2
- $\frac{dA}{dt}$ rate of change of the area $\frac{cm^2}{s}$
- $\frac{d\ell}{dt}$ rate of change of the side $\frac{cm}{s}$

Known: $\frac{dA}{dt} = 2\ \frac{cm^2}{s}$

Unknown: $\frac{d\ell}{dt}$ when $A = 25\ cm^2$

Equation: $A = \ell^2$

Differentiate: $\frac{d}{dt}[A] = \frac{d}{dt}[\ell^2]$

$$\frac{dA}{dt} = 2\ell \cdot \frac{d\ell}{dt} \quad \Rightarrow \quad \frac{d\ell}{dt} = \frac{1}{2\ell} \cdot \frac{dA}{dt}$$

When $A = 25\ cm^2$, $\ell = 5\ cm$, so $\frac{d\ell}{dt} = \frac{1}{2 \cdot (5\ cm)} \cdot \left(2\ \frac{cm^2}{s}\right) = \frac{1}{5}\ \frac{cm}{s}$

Each side is expanding at a rate of $\frac{1}{5}\ \frac{cm}{s}$ when the area is 25 cm^2.

2. (5 points)

Prove the Quotient Rule.

Want to show: $$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Two possible ways to prove the above formula:

<u>Strategy 1: use Product Rule and Chain Rule</u>

$$\begin{aligned}
\left[\frac{f(x)}{g(x)}\right]' &= \left[f(x)\cdot(g(x))^{-1}\right]' \\
&= f'(x)\cdot(g(x))^{-1} + f(x)\cdot\left[(g(x))^{-1}\right]' \\
&= \frac{f'(x)}{g(x)} + f(x)\cdot\left[(-1)\cdot(g(x))^{-2}\cdot g'(x)\right] \\
&= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{(g(x))^2} \\
&= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}
\end{aligned}$$

<u>Strategy 2: use Logarithmic Differentiation</u>

$$\begin{aligned}
y &= \frac{f(x)}{g(x)} \\
\ln(y) &= \ln\left(\frac{f(x)}{g(x)}\right) \\
\ln(y) &= \ln(f(x)) - \ln(g(x)) \\
\frac{d}{dx}\left[\ln(y)\right] &= \frac{d}{dx}\left[\ln(f(x)) - \ln(g(x))\right] \\
\frac{1}{y}\cdot y' &= \frac{1}{f(x)}\cdot f'(x) - \frac{1}{g(x)}\cdot g'(x) \\
y' &= y\cdot\left(\frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}\right) \\
y' &= \frac{f(x)}{g(x)}\cdot\left(\frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}\right) \\
y' &= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{(g(x))^2} \\
y' &= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}
\end{aligned}$$