Engineering Calculus I - MAC 2281 - Section 002

QUIZ VII

First Name: Last Name:

1. (4 points)

Compute the following limits. Show all work and **state any theorems or special limits used**.

- $\lim_{x\to\infty} x \cdot e^{-5x}$

$$\lim_{x\to\infty} x \cdot e^{-5x} = \lim_{x\to\infty} \frac{x}{e^{5x}} = \text{“}\frac{\infty}{\infty}\text{”}$$

because $\lim_{x\to\infty} x = \infty$ and $\lim_{x\to\infty} e^{5x} = \infty$

Since we obtained the indeterminate form “$\frac{\infty}{\infty}$”, we can apply L'Hospital's Rule:

$$\lim_{x\to\infty} x \cdot e^{-5x} = \lim_{x\to\infty} \frac{x}{e^{5x}} \underset{\text{L'H}}{=} \lim_{x\to\infty} \frac{(x)'}{(e^{5x})'} = \lim_{x\to\infty} \frac{1}{5 \cdot e^{5x}} = \text{“}\frac{1}{\infty}\text{”} = 0$$

- $\lim_{x\to 3} \frac{x^2-9}{x^2+9} = \frac{3^2-9}{3^2+9} = \frac{9-9}{9+9} = \frac{0}{18} = 0$

2. (4 points)

Prove that $\dfrac{d}{dx}\left[\arccos(x)\right] = -\dfrac{1}{\sqrt{1-x^2}}$ for $x \in (-1,1)$.

Let $y = \arccos(x)$. We have that

$$y = \arccos(x) \iff \cos(y) = x \text{ and } 0 \le y \le \pi$$

To prove that $\dfrac{dy}{dx} = -\dfrac{1}{\sqrt{1-x^2}}$, we differentiate $\cos(y) = x$ implicitly with respect to x:

$$\begin{aligned}\frac{d}{dx}\left[\cos(y)\right] &= \frac{d}{dx}\left[x\right]\\ -\sin(y)\cdot\frac{dy}{dx} &= 1\\ \frac{dy}{dx} &= -\frac{1}{\sin(y)}\end{aligned}$$

From the identity $\cos^2(y) + \sin^2(y) = 1$, we have $\sin^2(y) = 1 - \cos^2(y)$. Since $\sin(y) \ge 0$ when $0 \le y \le \pi$, we obtain

$$\sin(y) = \sqrt{1-\cos^2(y)} \underset{\substack{\uparrow\\ \cos(y)=x}}{=} \sqrt{1-x^2}$$

Hence

$$\frac{dy}{dx} = -\frac{1}{\sin(y)} = -\frac{1}{\sqrt{1-x^2}}$$

3. (2 points)

Compute $\dfrac{d}{dx}\left[x^3\cdot\arcsin(3x)\right]$. Show sufficient work to **communicate** your process.

$$\begin{aligned}\frac{d}{dx}\left[x^3\cdot\arcsin(3x)\right] &= \frac{d}{dx}\left[x^3\right]\cdot\arcsin(3x) + x^3\cdot\frac{d}{dx}\left[\arcsin(3x)\right]\\ &= 3x^2\cdot\arcsin(3x) + x^3\cdot\frac{1}{\sqrt{1-(3x)^2}}\cdot\frac{d}{dx}[3x]\\ &= 3x^2\cdot\arcsin(3x) + x^3\cdot\frac{1}{\sqrt{1-9x^2}}\cdot 3\\ &= 3x^2\cdot\arcsin(3x) + \frac{3x^3}{\sqrt{1-9x^2}}\end{aligned}$$