Engineering Calculus I - MAC 2281 - Section 002

QUIZ VIII

First Name: Last Name:

1. (3 points)

State the Extreme Value Theorem.

If f is continuous on $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$

2. (3 points)

State the Mean Value Theorem.

If f is continuous on $[a, b]$ and differentiable on (a, b), then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

3. (4 points)

Showing **all your work**, find the absolute maximum and absolute minimum of the function $h(x) = -2x^3 - 3x^2 + 12x + 5$ on $[-3, 3]$.

(1) Find the critical numbers of h in $(-3, 3)$ and their corresponding values:

$$h'(x) = -6x^2 - 6x + 12 = -6(x^2 + x - 2) = -6(x + 2)(x - 1)$$

Since $h'(x)$ exists for all $x \in (-3, 3)$, the critical numbers of h in $(-3, 3)$ are given by the numbers $c \in (-3, 3)$ such that $h'(c) = 0$. We have that $h'(x) = 0$ when $x = -2$ or $x = 1$, and the numbers -2 and 1 both belong to the interval $(-3, 3)$. Thus the critical numbers of h in $(-3, 3)$ are -2 and 1. The values of h at these critical numbers are

$$h(-2) = -2 \cdot (-2)^3 - 3 \cdot (-2)^2 + 12 \cdot (-2) + 5 = -15$$
$$h(1) = -2 \cdot 1^3 - 3 \cdot 1^2 + 12 \cdot 1 + 5 = 12$$

(2) Compute the values of h at the endpoints of $[-3, 3]$:

$$h(-3) = -2 \cdot (-3)^3 - 3 \cdot (-3)^2 + 12 \cdot (-3) + 5 = -4$$
$$h(3) = -2 \cdot 3^3 - 3 \cdot 3^2 + 12 \cdot 3 + 5 = -40$$

(3) Compare the values obtained in Step (1) and Step (2):

$$h(-2) = -15$$
$$h(1) = 12 \leftarrow \text{absolute maximum}$$
$$h(-3) = -4$$
$$h(3) = -40 \leftarrow \text{absolute minimum}$$